

Enhancement of $B(E2)\uparrow$ and low excitation of the second 0^+ state near $N = 40$ in Ge isotopes

M. Hasegawa^a, T. Mizusaki^b, K. Kaneko^c, Y. Sun^{d,e,f}

^a*Laboratory of Physics, Fukuoka Dental College, Fukuoka 814-0193, Japan*

^b*Institute of Natural Sciences, Senshu University, Tokyo 101-8425, Japan*

^c*Department of Physics, Kyushu Sangyo University, Fukuoka 813-8503, Japan*

^d*Department of Physics and Joint Institute for Nuclear Astrophysics, University of Notre Dame, Notre Dame, IN 46556, USA*

^e*Department of Physics, Tsinghua University, Beijing 100084, P. R. China*

^f*Department of Physics, Xuzhou Normal University, Xuzhou, Jiangsu 221009, P. R. China*

Abstract

The long-standing problem of dramatic structure change near $N = 40$ in Ge isotopes is investigated by means of large-scale shell model calculation. The analysis of simulated calculations suggests a possible understanding of the problem in terms of rapid increase in the $g_{9/2}$ proton and neutron occupation. The observed variation in excitation of the second 0^+ state in $^{70,72,74}\text{Ge}$ appears to correlate closely with the $g_{9/2}$ occupations induced by strong proton-neutron interactions. The enhancement of the $g_{9/2}$ occupancies is probably due to correlations in the $1g2d3s$ shell.

Key words: $^{70,72,74}\text{Ge}$, structure change, $B(E2)$, second 0^+ energy, shell model

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1 Introduction

Experimental information on nuclei far from stability constitutes a challenging test for the applicability of nuclear models and serves as a guidance for improvement of the models. Radioactive ion beam facility plays a vital role in gathering such information. In a recent experiment at the Oak Ridge HRIBF, Padilla-Rodal *et al.* measured $B(E2; 0_1^+ \rightarrow 2_1^+)$ in $^{78,80,82}\text{Ge}$ [1], which provides a complete view of $B(E2)\uparrow$ data extended to the neutron-rich region, reaching the major shell closure at $N = 50$. It was also demonstrated [1] that the observed $B(E2)$ values in $^{78,80,82}\text{Ge}$ can be reproduced by using a shell

model with the model space $(p_{3/2}, f_{5/2}, p_{1/2}, g_{9/2})$ for both protons and neutrons. Hence this work showed that the structure change in the exotic mass region could be microscopically studied by means of large-scale shell model calculation, which has become available only recently. Large-scale shell model calculations have been applied to the study of $N = 40$ magicity in Refs. [2,3,4,5].

The work of Padilla-Rodal *et al.* has stimulated our interest in the long-standing problem of structure change along the Ge isotopic chain, particularly in the vicinity of $N = 40$. The experimental $B(E2)\uparrow$ data shown in Fig. 4 of Ref. [1] suggest that ^{68}Ni can be regarded as a quasi-doubly-closed-shell nucleus, but the nature of the $N = 40$ subshell closure is gradually destroyed as the proton number outside the $f_{7/2}$ orbit increases. The variation of $B(E2)\uparrow$ along the N axis reminds us of the old problem, much discussed in the 1970's and 1980's, of the structure change from the $N = 38$ to $N = 42$ Ge isotope. References were listed, for instance, in Ref. [6] published in 1987. A notable irregularity at $N = 40$ was observed in the (p, t) and (t, p) reaction cross sections [7,8,9], which is characterized by a drastic drop of the second 0^+ excitation energy in ^{72}Ge . This is an unusually low excited 0^+ state, showing a similar character as that of typical doubly-closed-shell nuclei. At $N = 40$, the drop of $E_x(0_2^+)$ appears in coincidence with a remarkable enhancement of $B(E2)\uparrow$. These experimental data suggest that the structure of ^{72}Ge with $N = 40$ is very different from that of ^{68}Ni studied in Refs. [2,3,4].

The structure changes in Ge isotopes were discussed in terms of pairing correlation and shape change (or shape coexistence) by using various collective models [10,11,12,13]. The interacting boson model [14] reproduced the observed energy levels and $B(E2)$ values for $^{68-76}\text{Ge}$. However, a satisfactory microscopic explanation has not been seen. Most of the early studies in the literature limited their discussion on the configuration change within the fp shell. Contributions from the $g_{9/2}$ orbit were very little mentioned. These early studies are insufficient according to the new knowledge on the $N = 40$ magicity obtained in Refs. [2,3,4,5], which has emphasized that nucleons in the $g_{9/2}$ orbit play a crucial role in determining the structure of the $N = 40$ isotones. Thus, as a necessary requirement, a microscopic study for the structure in Ge isotopes should include the $g_{9/2}$ orbit.

2 What problem exists?

In the previous publications [15,16,17], we have demonstrated the feasibility of a shell model description for $^{64-68}\text{Ge}$ (including odd-mass isotopes), and for some other neighboring nuclei of the mass region. In these studies, large-scale shell model calculations (up to dimension 2×10^8) were performed in the model

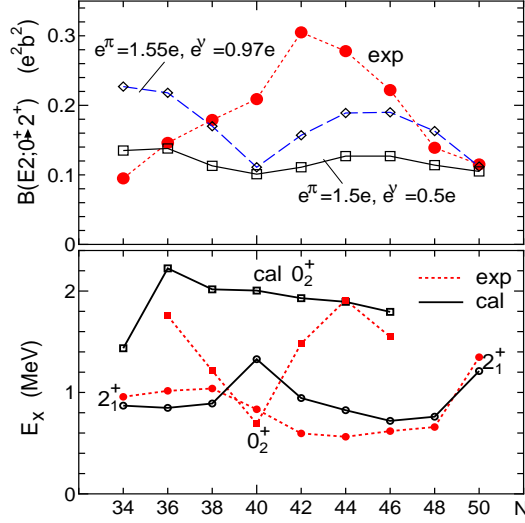


Fig. 1. Calculated and experimental values of $B(E2)\uparrow$, $E_x(2_1^+)$, and $E_x(0_2^+)$ for the Ge isotopes from $N = 34$ to 50. Calculated $B(E2)\uparrow$ with two sets of effective charge are shown for comparison.

space $(p_{3/2}, f_{5/2}, p_{1/2}, g_{9/2})$, the same model space as that of Ref. [1]. The aim of the present article is twofold: to fill the gap in the Ge isotopic chain that the previous shell model calculations have neglected, and to look for possible structure reasons that may explain the unsolved puzzle in the middle of the chain around $N = 40$.

The extended pairing plus quadrupole ($P+QQ$) Hamiltonian [15] is employed in the present work. This Hamiltonian has recently been proposed and tested through several shell model applications. It should be stressed that this isospin-invariant Hamiltonian includes strong proton-neutron interactions in both $T = 0$ and $T = 1$ channel. The single-particle energies in our model are $\varepsilon_{p3/2}=0.00$, $\varepsilon_{f5/2}=0.77$, $\varepsilon_{p1/2}=1.11$, and $\varepsilon_{g9/2}=2.50$ (in MeV). In the present work that deals with a long isotopic chain, we use the A -dependent interaction strengths: $g_0 = 0.27(64/A)$ MeV for the monopole pairing force, $\chi_2 = 0.25(64/A)^{5/3}$ MeV for the quadrupole-quadrupole (QQ) force, and $\chi_3 = 0.05(64/A)^2$ MeV for the octupole-octupole (OO) force. These are the same parameters as those in Ref. [15]. The Hamiltonian contains also five monopole correction terms (see Ref. [15]). The effective charges are $e_{eff}^\pi = 1.5e$ and $e_{eff}^\nu = 0.5e$.

Our results are presented in Fig. 1, where calculated $B(E2)\uparrow$, $E_x(2_1^+)$, and $E_x(0_2^+)$ for the Ge isotopes from ^{66}Ge ($N = 34$) to ^{82}Ge ($N = 50$) are compared with data. The calculation reasonably reproduces $B(E2)\uparrow$ and $E_x(0_2^+)$ at both ends of the isotopic chain with $N = 34, 36$ and $N = 48, 50$, but fails to describe those in between. Especially, the calculation could not reproduce the increasing trend of $B(E2)\uparrow$ over $N = 40$ and the drop of $E_x(0_2^+)$ around this neutron number. We can get an improved agreement with the $B(E2)\uparrow$ data for ^{70}Ge and ^{78}Ge (see Fig. 1) if we enlarge the effective charges to $e_{eff}^\pi = 1.55e$

and $e_{eff}^\nu = 0.97e$ ($e_{eff}^\nu = 0.97e$ is taken from Ref. [1] and $e_{eff}^\pi = 1.55e$ is fixed so as to reproduce $B(E2)\uparrow$ in ^{82}Ge). However, the use of larger effective charges destroys the good agreement for ^{66}Ge and ^{68}Ge already obtained in Ref. [15]. Moreover, the rise in effective charges does not seem to have any positive effect on enhancing $B(E2)\uparrow$ at $N = 40$. It is thus clear that the problem must stem from deeper structure reasons, which cannot be resolved simply by a global fit through effective charges.

The above results indicate an inadequacy either in the interaction employed in the calculation, or in the model space for $^{70-76}\text{Ge}$ in question. At present, there is no available effective interaction better than the extended $P + QQ$ interaction for Ge isotopes in the model space $(p_{3/2}, f_{5/2}, p_{1/2}, g_{9/2})$. We thus proceed our study with the extended $P + QQ$ interaction and look for any possible reasons for this discrepancy.

The enhancement of $B(E2)\uparrow$ in the $N = 40$ isotones from ^{68}Ni to ^{78}Sr has been qualitatively explained by Langanke *et al.* [5] using the Shell Model Monte Carlo (SMMC) approach, where a $P + QQ$ Hamiltonian (with no monopole terms) is used but their model space is larger, which includes also the $1g_{7/2}2d_{3/2}$ shell. It must be noted that the SMMC calculation gives only total $B(E2)$ strength to excited 2^+ states and does not give the energy of the 0_2^+ state. The SMMC calculation predicted a large neutron occupation number in the $g_{9/2}$ orbit ($\langle n_{g_{9/2}}^\nu \rangle > 2$), in contrast to our small value ($\langle n_{g_{9/2}}^\nu \rangle \sim 0.23$) obtained for ^{72}Ge . This gives us a hint that inclusion of the $1g_{7/2}2d_{3/2}$ orbits may significantly enhance the role of the $g_{9/2}$ orbit through the pairing and quadrupole correlations when the Fermi level moves toward the $1g_{7/2}2d_{3/2}$ shell. The deficiency in our calculation may be due to the insufficient model space for the nuclei where neutrons start to occupy the $p_{1/2}$ and $g_{9/2}$ orbits.

The $g_{9/2}$ occupancy of neutrons is investigated in ^{68}Ni with the ordinary shell model [2,3], in which the $g_{9/2}$ orbit is excluded from the model space for protons. The value $\langle n_{g_{9/2}}^\nu \rangle \sim 1.2$ obtained in Ref. [3] is considerably different from the SMMC result $\langle n_{g_{9/2}}^\nu \rangle \sim 2.2$ [5]. In another recent work [18], we have investigated ^{68}Ni in the neutron model space $(p_{3/2}, f_{5/2}, p_{1/2}, g_{9/2})$ assuming a ^{56}Ni core. The shell model for neutrons also gives the value $\langle n_{g_{9/2}}^\nu \rangle \sim 1.2$, similar to that in Ref. [3]. These shell model calculations show that the ground state of this nucleus remains a transitional character to a superfluid phase. It is thus interesting to investigate whether the delicate structure changes from the $N = 38$ to $N = 42$ Ge isotopes are caused by strong pairing correlations.

3 Renormalization approach within the truncated space

We continue our study within the $2p1f_{5/2}1g_{9/2}$ space as this model space is at present the largest possible for a full shell model diagonalization. Within this truncated space, contributions from the $2d1g_{7/2}3s$ orbits must be expressed in terms of effective interaction. We have made testing calculations with different monopole corrections, which effectively change the neutron $g_{9/2}$ occupation. However, moderate modifications for the monopole corrections cannot produce a significant structure change near $N = 40$. On the other hand, it is expected that the effects of the $2d1g_{7/2}3s$ orbits tend to strengthen the pairing and quadrupole correlations. We therefore consider to enlarge the quadrupole matrix element $Q(g_{9/2}g_{9/2})$ to mimic any effect from the missing $2d1g_{7/2}3s$ orbits. Note that the quadrupole matrix elements between the $1g_{9/2}$ and $2d_{5/2}1g_{7/2}$ orbits are nonzero and those between the fp and gds shells are zero (the $2d_{5/2}$ orbit nearest to $1g_{9/2}$ has the largest quadrupole matrix element with $1g_{9/2}$).

Based on the above considerations, we write the effective quadrupole matrix element for the $g_{9/2}$ orbit by multiplying a factor η

$$Q_{eff}(g_{9/2}g_{9/2}) = \eta Q(g_{9/2}g_{9/2}). \quad (1)$$

Physically, an $\eta > 1$ factor strengthens the QQ force with respect to the $g_{9/2}$ orbit, which is consistent with the expectation that the Ge isotopes become deformed at the beginning of the $g_{9/2}$ orbit and that a strong QQ force contributes to lowering of the $g_{9/2}$ orbit. A stronger $Q(g_{9/2}g_{9/2})$ increases also the effective charges selectively for the $g_{9/2}$ nucleons. In our shell model treatment, we transform the QQ force into multipole-pairing-type interactions in the particle-particle channel. Thus effectively, a stronger QQ force with respect to the $g_{9/2}$ orbit enhances also the pairing correlations, which allows more nucleons to jump to the $g_{9/2}$ orbit.

The effects from the $2d1g_{7/2}3s$ orbits are considered to be isotope dependent and should become larger as the Fermi level moves up. Thus in our trial calculations, we assume an empirical expression for η , $\eta = 1 + 0.05(N - 32)$. With these considerations, the calculations indeed achieve a great improvement in excitation energies $E_x(2_1^+)$ and $E_x(0_2^+)$, especially for $^{66-74}\text{Ge}$. The calculated $B(E2)\uparrow$ values remain good at the beginning of the chain ($N = 34$ and 36 , because the configurations are almost exhausted by the fp shell), and are enhanced at the end with $N = 46 - 50$. These improvements support the use of effective quadrupole matrix element $Q_{eff}(g_{9/2}g_{9/2})$. However, $B(E2)\uparrow$ for $^{72-76}\text{Ge}$ are again not enhanced. This indicates that the microscopic content in the wave-functions has not been improved. As a matter of fact, the neutron and proton occupation numbers $\langle n_{g_{9/2}}^\nu \rangle \sim 0.62$ and $\langle n_{g_{9/2}}^\pi \rangle \sim 0.07$ obtained for ^{72}Ge ($N = 40$) by using $Q_{eff}(g_{9/2}g_{9/2})$ cannot bear a comparison with

the large values $\langle n_{g9/2}^\nu \rangle \sim 3$ and $\langle n_{g9/2}^\pi \rangle \sim 0.2$ reported in Ref. [5]. The large $B(E2)^\uparrow$ value in ^{72}Ge corresponds to a large occupation of the $g_{9/2}$ orbit, which must be due to the pairing correlations in addition to the QQ force.

In fact, the unusually low energy of the second 0^+ state in ^{72}Ge implies a delicate balance between the pairing correlations and the energy gap $\varepsilon_{g9/2} - \varepsilon_{p1/2}$. It has been pointed out by Schiffer *et al.* [19] and Utsuno *et al.* [20] that the “effective energy” of the intruder states changes rapidly around a closed subshell in neutron-rich nuclei, and these changes were discussed in terms of the spin-orbit interaction or the spin-isospin dependence of nucleon-nucleon interaction. In our case, the change of the effective energy gap $\varepsilon_{g9/2} - \varepsilon_{p1/2}$ could be attributed to the pairing correlations in the $1g2d3s$ shell. A good effective Hamiltonian in a full shell model should be able to describe the balance microscopically. In the present work with the truncated model space, we try to understand the problem by simulating the underlying physics. Namely, we adjust the energy gap $\varepsilon_{g9/2} - \varepsilon_{p1/2}$ and study any resultant change

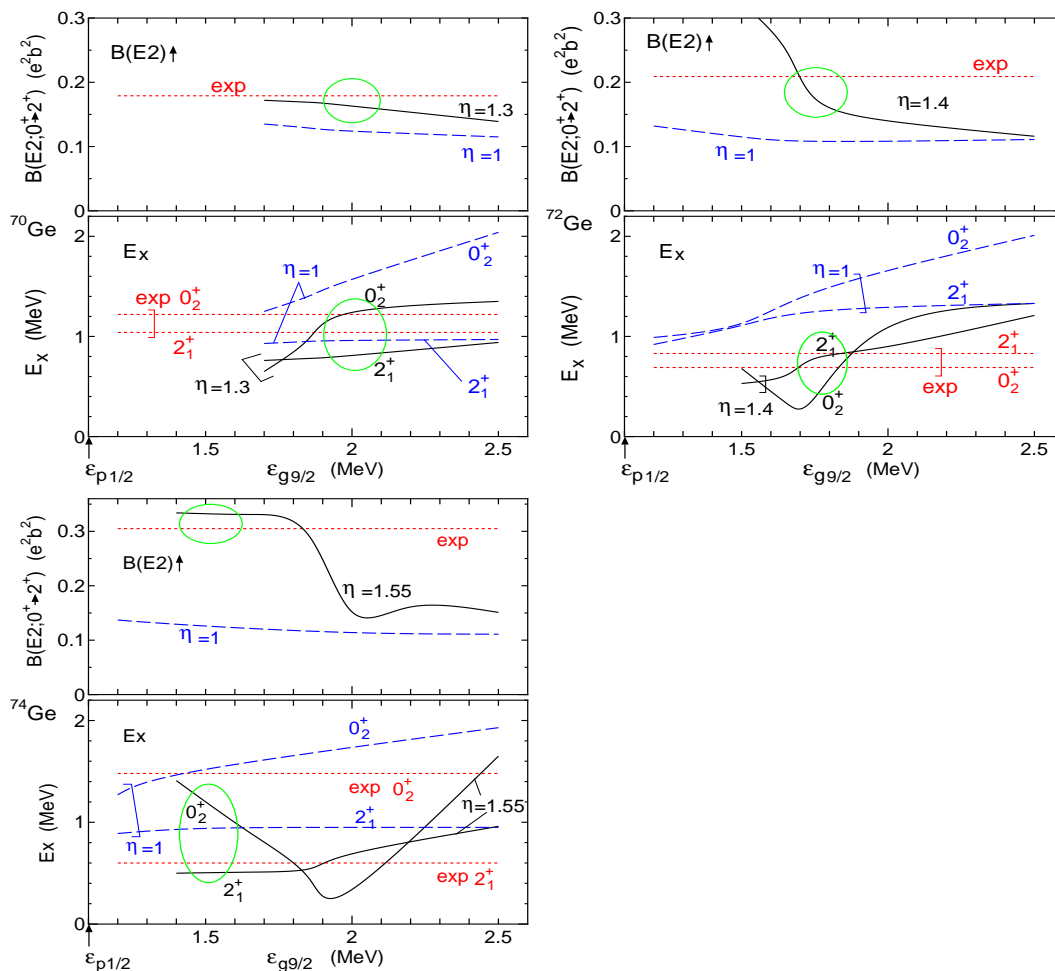


Fig. 2. Variations of $B(E2)^\uparrow$, $E_x(2_1^+)$, and $E_x(0_2^+)$ in $^{70,72,74}\text{Ge}$ as $\varepsilon_{g9/2}$ approaches $\varepsilon_{p1/2}$.

in $B(E2)\uparrow$ and $E_x(0_2^+)$.

We study variations of $B(E2)\uparrow$ and $E_x(0_2^+)$ in $^{66-74}\text{Ge}$ as a function of single-particle separation energy between the $g_{9/2}$ and $p_{1/2}$ orbits. In the calculation, we use the enhancement factor $\eta = 1 + 0.05(N - 32)$ for $^{66-72}\text{Ge}$, and a slightly larger one $\eta = 1.55$ for ^{74}Ge . Interesting results are obtained as shown in Fig. 2. The circled areas represent the best choices in parameter that can describe the observed $B(E2)\uparrow$, and at the same time, give correct sequences of 2_1^+ and 0_2^+ in $^{70-74}\text{Ge}$. Figure 2 indicates clearly the necessity of a rapid lowering of the single-particle energy $\varepsilon_{g9/2}$ from the original value 2.5 MeV in $^{66-68}\text{Ge}$, to ~ 2.0 MeV in ^{70}Ge , to ~ 1.75 MeV in ^{72}Ge , and to ~ 1.5 MeV in ^{74}Ge . Thus the calculation demonstrates that the observed enhancement in $B(E2)\uparrow$ and variation in $E_x(0_2^+)$ correlate closely with the single-particle energy separation. Data cannot be understood unless the effective energy gap $\varepsilon_{g9/2} - \varepsilon_{p1/2}$ near $N = 40$ is set to be small. On the other hand, the calculated $B(E2)\uparrow$ in ^{72}Ge and ^{74}Ge cannot reach the observed large values unless the enlarged matrix element $Q_{eff}(g_{9/2}g_{9/2})$ is used. In particular, the correct sequence of the 0_2^+ and 2_1^+ states in ^{74}Ge cannot be obtained without narrowing the effective gap $\varepsilon_{g9/2} - \varepsilon_{p1/2}$ to a sensitive region in the parameter space, which implies an extremely delicate balance between the pairing correlations and the energy separation $\varepsilon_{g9/2} - \varepsilon_{p1/2}$.

Although a meaningful parameter choice can be found for the isotopes up to ^{74}Ge , it is difficult to go beyond them and find good parameters that can simultaneously reproduce $B(E2)\uparrow$ and $E_x(0_2^+)$ for ^{76}Ge , as illustrated in Fig. 3. This indicates limitations in our renormalization approach within the truncated model space missing $2d1g_{7/2}3s$ orbits above $g_{9/2}$. As we shall see in Sect. IV, a small energy separation $\varepsilon_{g9/2} - \varepsilon_{p1/2}$ causes nucleons to jump to the $g_{9/2}$ orbit easily, and with more nucleons in the $g_{9/2}$ orbit the use of $Q_{eff}(g_{9/2}g_{9/2})$ enhances $B(E2)$ values in $^{70-74}\text{Ge}$. However, the situation seems to be dif-

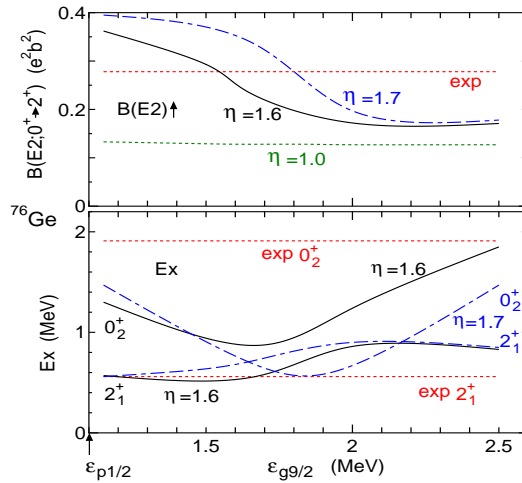


Fig. 3. Variations of $B(E2)\uparrow$, $E_x(2_1^+)$, and $E_x(0_2^+)$ in ^{76}Ge as $\varepsilon_{g9/2}$ approaches $\varepsilon_{p1/2}$.

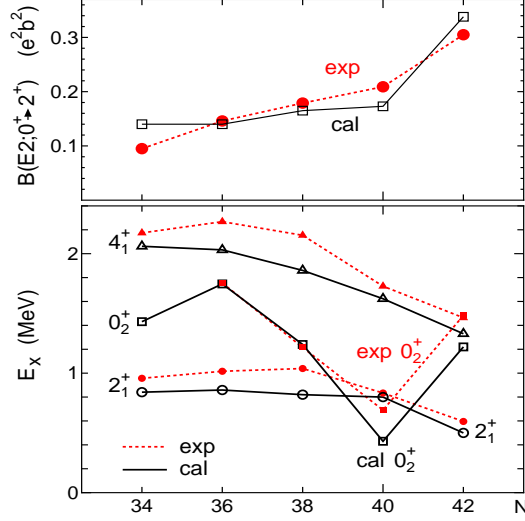


Fig. 4. Comparison of improved calculation for $B(E2)_{\uparrow}$, $E_x(2_1^+)$, and $E_x(0_2^+)$ with experimental data.

ferent between $^{70-74}\text{Ge}$ and ^{76}Ge . In $^{70-74}\text{Ge}$, the $g_{9/2}$ orbit is occupied by at most four neutrons. In ^{76}Ge , however, about half of the neutron $g_{9/2}$ orbit is occupied. With an additional neutron pair in the $g_{9/2}$ orbit, $Q_{eff}(g_{9/2}g_{9/2})$ enhances $B(E2)$ exaggeratedly. In such a case, our renormalization approach fails to work properly. It would be necessary for the description of the $N > 40$ isotopes to explicitly include upper orbits such as $d_{5/2}$, as suggested in Ref. [2]. Below, we continue our discussion but focus the attention on the structure change in ^{70}Ge , ^{72}Ge , and ^{74}Ge only.

With the above adjustment in $\varepsilon_{g_{9/2}} - \varepsilon_{p_{1/2}}$ and the use of effective quadrupole matrix element $Q_{eff}(g_{9/2}g_{9/2})$, we are able to reproduce correctly all the observed variations in $B(E2)_{\uparrow}$, $E_x(0_2^+)$, and $E_x(2_1^+)$ for $^{66-74}\text{Ge}$, as summarized in Fig. 4. Here, the energy separation $\varepsilon_{g_{9/2}} - \varepsilon_{p_{1/2}}$ is not required to change for the ^{66}Ge and ^{68}Ge calculations if $Q_{eff}(g_{9/2}g_{9/2})$ with $\eta = 1 + 0.05(N - 32)$ is used. Figure 4 shows that the variation trend of 4_1^+ energy is also correctly reproduced. Of course, our simulated calculation does not provide a final answer to the problem, but is quite suggestive. It should be stressed that the set of circles in Fig. 2 is the sole selection in parameter that can describe the experimental data.

4 What changes in structure?

The important information extracted from the above discussions is that in order to describe all the observed quantities in Ge isotopes, it is necessary to have enough $g_{9/2}$ contributions in the wave-functions. Next, let us study how the $g_{9/2}$ proton and neutron occupations correlate with the observations

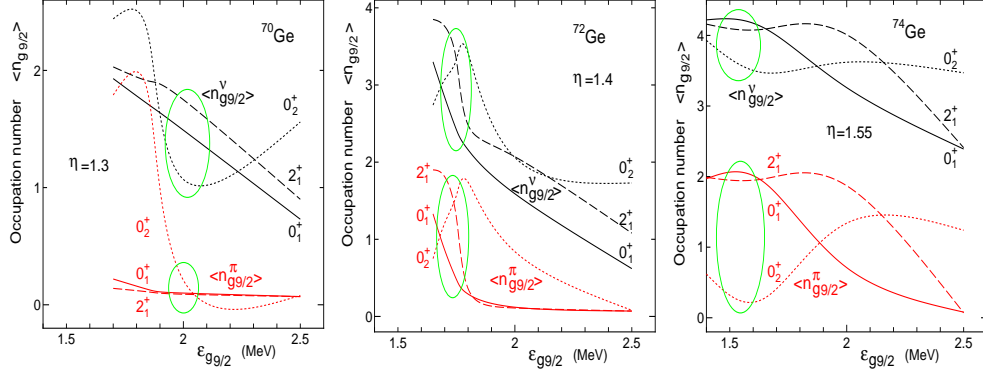


Fig. 5. (Color online) Variations of $\langle n_{g9/2}^\pi \rangle$ and $\langle n_{g9/2}^\nu \rangle$ in $^{70,72,74}\text{Ge}$ as $\epsilon_{g9/2}$ approaches $\epsilon_{p1/2}$.

and how they vary in $^{70,72,74}\text{Ge}$. We calculate occupation numbers $\langle n_{g9/2} \rangle$ for protons and neutrons as functions of the energy separation $\epsilon_{g9/2} - \epsilon_{p1/2}$, and focus our discussion on the results in the circled area that have reproduced data. Figure 5 shows that in ^{70}Ge , a considerable number of neutrons occupy the $g_{9/2}$ orbit ($\langle n_{g9/2}^\nu \rangle \gtrsim 1$) while protons are scarcely found in this orbit. For the ground state of ^{72}Ge , the occupation numbers become $\langle n_{g9/2}^\pi \rangle \sim 0.5$ and $\langle n_{g9/2}^\nu \rangle \sim 2.5$. These occupation numbers are consistent with those of Ref. [5], though our value $\langle n_{g9/2}^\pi \rangle \sim 0.5$ is larger. The result indicates a superfluid state for neutrons in ^{72}Ge , in contrast to the narrow subshell closure in ^{68}Ni [2,3,18]. It is notable that in the circled area for ^{72}Ge , the coupling between the 0_1^+ and 0_2^+ states becomes significant and a considerable number of protons occupy the $g_{9/2}$ orbit, which shows a qualitative difference from the structure in ^{70}Ge . The structure in ^{74}Ge again differs qualitatively from ^{72}Ge . As $\epsilon_{g9/2}$ approaches $\epsilon_{p1/2}$, the proton occupation numbers $\langle n_{g9/2}^\pi \rangle$ for 0_1^+ and 0_2^+ are reversed. For $\epsilon_{g9/2} \lesssim 1.75$ MeV with which the calculated $B(E2) \uparrow$ are comparable with data, more than four neutrons occupy the $g_{9/2}$ orbit and protons occupy this orbit also considerably, in sharp contrast to the situation in ^{70}Ge .

Thus the structure differences appearing in our calculation for the two lowest 0^+ states, 0_1^+ and 0_2^+ , may offer a new explanation for the old problem observed in (p, t) and (t, p) reactions. Our simulated calculation predicts that the proton structure in the $g_{9/2}$ orbit is very different among the states in $^{70,72,74}\text{Ge}$. The proton $g_{9/2}$ occupation changes from a negligible amount in ^{70}Ge to a considerable amount in $^{72,74}\text{Ge}$, and in addition, with a different distribution in the 0_1^+ and 0_2^+ states.

Obviously, these results cannot be obtained without the presence of strong proton-neutron correlations in the model, which are important for nuclei where protons and neutrons occupy the same shell. The energy difference $\epsilon_{g9/2} - \epsilon_{p1/2} = 1.34$ MeV used in our calculation, which is adopted so as to well describe $^{66-68}\text{Ge}$ [15] and also consistent with the value used in Ref. [21], is rather small. Still, for the isotopes up to ^{70}Ge , the proton occupation number $\langle n_{g9/2}^\pi \rangle$

remains small, in contrast to the strong configuration mixing within the fp shell including the $p_{1/2}$ orbit. In this sense, for protons when $N \leq 38$, there exists a gap between the $pf_{5/2}$ shell and $g_{9/2}$. However, as clearly seen from Fig. 5, the proton occupation number $\langle n_{g_{9/2}}^\pi \rangle$, together with the neutron occupation number $\langle n_{g_{9/2}}^\nu \rangle$, increases suddenly in ^{72}Ge and ^{74}Ge . This “resonance” effect of $\langle n_{g_{9/2}}^\pi \rangle$ and $\langle n_{g_{9/2}}^\nu \rangle$ can be best attributed to strong proton-neutron correlations. Similar effects are obtained in the SMMC calculations for the $N = 40$ isotones [5], where a larger energy difference $\varepsilon_{g_{9/2}} - \varepsilon_{p_{1/2}} = 2.44$ (or 1.97) MeV is used but upper $2d_{1/2}3s$ orbits above $g_{9/2}$ are included in the model space. A considerably large number of $\langle n_{g_{9/2}}^\pi \rangle$ starts from ^{72}Ge (where Z is only equal to 32 and N takes the magic number 40). We may thus conclude that the proton $g_{9/2}$ occupancy must be caused by the proton-neutron correlations in the gds shell. Our treatment with small $\varepsilon_{g_{9/2}} - \varepsilon_{p_{1/2}}$ and large $Q(g_{9/2}g_{9/2})$ is considered to mimic the effects, since the mechanism is not properly included in the model. It should be noted in this regard that our isospin-invariant Hamiltonian treats dynamically the strong proton-neutron interactions.

5 Concluding remarks

In conclusion, inspired by the recent experimental advances in $B(E2)$ measurement for neutron-rich Ge isotopes, we have carried out a systematical shell model calculation for $^{66-82}\text{Ge}$ in the model space $(p_{3/2}, f_{5/2}, p_{1/2}, g_{9/2})$. The calculations have shown that the strong enhancement of $B(E2)^\uparrow$ and the unusually low excitation of the second 0^+ state near $N = 40$ can be explained only with sufficient occupation of protons and neutrons in the $g_{9/2}$ orbit. The simulated calculations that mimic such effects have suggested a possible understanding of the structure change from ^{70}Ge to ^{74}Ge in terms of rapid increase in the number of $g_{9/2}$ protons and neutrons. This could be the source for the long-standing problem that has not yet been understood. The isotopic dependence of the $g_{9/2}$ proton occupation must have an origin of strong proton-neutron interactions.

In principle, it is a very difficult question whether all these effects can be expressed in an effective interaction. The calculations in Ref. [2] pointed out that the $d_{5/2}$ orbit above the $g_{9/2}$ makes a considerable contribution when $N > 36$ in Cr and Fe isotopes apart from the proton magic number $Z = 28$. The SMMC calculations in the $1f2p - 1g2d3s$ space [5] also suggested an important role of the $d_{5/2}$ orbit in the enhancement of $g_{9/2}$ occupancies in those nuclei that have sufficient number of protons in the upper $1f2p$ shell. According to the extended $P+QQ$ model, we can suggest that the pairing and quadrupole-quadrupole correlations in the major shell $1g2d3s$ make it easier for protons and neutrons to be excited from the $1f2p$ shell to the $g_{9/2}$ orbit.

The present investigation with enlarged $Q(g_{9/2}g_{9/2})$ and decreased energy gap $\varepsilon_{g9/2} - \varepsilon_{p1/2}$ is along the line of this thought. The problem may be thoroughly explained if shell model calculations are performed in a two-major-shell-space including both $1f2p$ and $1g2d3s$.

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